Overview of Molecular Modeling
Molecular Modeling: Introduction

What is Molecular Modeling?

Molecular Modeling is concerned with the description of the atomic and molecular interactions that govern *microscopic* and *macroscopic* behaviors of physical systems.

What is it good for?

The essence of molecular modeling resides in the connection between the *microscopic* world and the *macroscopic* world provided by the theory of statistical mechanics.

Macroscopic observable
(Solvation energy, affinity between two proteins, H-H distance, conformation, ...)

Average of observable over selected microscopic states
Dipole Moment of Carbon Monoxide

Value of Dipole Moment

Time in Years

1950  1960  1970

+0.5

-0.5

Experiment

Theory
Fig. 1. Molecular models, simulation and experiment.
<table>
<thead>
<tr>
<th>QUANTUM MECHANICS</th>
<th>CRystalline Solid State</th>
<th>LIQUID STATE Macromolecules</th>
<th>GAS PHASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>possible</td>
<td>still impossible</td>
<td>possible</td>
<td></td>
</tr>
<tr>
<td>CLASSICAL STATISTICAL MECHANICS</td>
<td>easy</td>
<td>computer simulation</td>
<td>trivial</td>
</tr>
</tbody>
</table>

Fig. 1. Classification of molecular systems. Systems in the shaded area are amenable to treatment by computer simulation.
Fig. 3. Choice of molecular model, force field and sample size depends on 1) the property one is interested in (space to be searched), 2) required accuracy of the prediction, 3) the available computing power to generate the ensemble.
<table>
<thead>
<tr>
<th>Motion</th>
<th>Spatial extent (nm)</th>
<th>Amplitude (nm)</th>
<th>Log of characteristic time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative vibration of bonded atoms</td>
<td>0.2 to 0.5</td>
<td>0.001 to 0.01</td>
<td>-14 to -13</td>
</tr>
<tr>
<td>Longitudinal motions of bases in double helices (nucleic acids)</td>
<td>0.5</td>
<td>0.01</td>
<td>-14 to -13</td>
</tr>
<tr>
<td>Lateral motions of bases in double helices (nucleic acids)</td>
<td>0.5</td>
<td>0.1</td>
<td>-13 to -12</td>
</tr>
<tr>
<td>Global stretching (nucleic acids)</td>
<td>1 to 30</td>
<td>0.03 to 0.3</td>
<td>-13 to -11</td>
</tr>
<tr>
<td>Global twisting (nucleic acids)</td>
<td>1 to 30</td>
<td>0.1 to 1.0</td>
<td>-13 to -11</td>
</tr>
<tr>
<td>Elastic vibration of globular region</td>
<td>1 to 2</td>
<td>0.005 to 0.05</td>
<td>-12 to -11</td>
</tr>
<tr>
<td>Sugar repuckering (nucleic acids)</td>
<td>0.5</td>
<td>0.2</td>
<td>-12 to -9</td>
</tr>
<tr>
<td>Rotation of sidechains at surface (protein)</td>
<td>0.5 to 1</td>
<td>0.5 to 1</td>
<td>-11 to -10</td>
</tr>
<tr>
<td>Torsional libration of buried groups</td>
<td>0.5 to 1</td>
<td>0.05</td>
<td>-11 to -9</td>
</tr>
<tr>
<td>Relative motion of different globular regions (hinge bending)</td>
<td>1 to 2</td>
<td>0.1 to 0.5</td>
<td>-11 to -7</td>
</tr>
<tr>
<td>Global bending (nucleic acids)</td>
<td>10 to 100</td>
<td>5 to 20</td>
<td>-10 to -7</td>
</tr>
<tr>
<td>Rotation of medium-sized sidechains in interior (protein)</td>
<td>0.5</td>
<td>0.5</td>
<td>-4 to 0</td>
</tr>
<tr>
<td>Allosteric transitions</td>
<td>0.5 to 4</td>
<td>0.1 to 0.5</td>
<td>-5 to 0</td>
</tr>
<tr>
<td>Local denaturation</td>
<td>0.5 to 1</td>
<td>0.5 to 1</td>
<td>-5 to +1</td>
</tr>
</tbody>
</table>
Molecular Models:

Physical

- Framework
- Space Filling

Mathematical

- Quantum Mechanics
- Classical (Empirical) Potential Functions
- Statistical or Database Derived
Introduction to molecular surfaces

Definitions:
- **Van der Waals**: ensemble of van der Waals sphere centered at each atom
- **Connolly**: ensemble of contact points between probe and vdw spheres
- **Solvent**: ensemble of probe sphere centers
Examples of molecular surfaces

Van der Waals

Connolly (Contact)

Solvent accessible
EXAMPLE: \textsc{water}

\textsc{stillinger} (ST2)

\[ q = 0.2357 \]
\[ E_{LT} = 0.31 \cdot \left( \frac{\sigma}{R} \right)^2 - 0.31 \cdot \left( \frac{\sigma}{R} \right)^6, \quad \sigma = 3.10 \text{Å} \]

\textsc{jorgensen} (TIP4P)

\[ 0-x = 0.15 \text{Å} \]
\[ q = 0.52 \]
\[ R = 0.9572 \quad \theta = 104.52 \]
\[ E_{LT} = 600/R^2 - 610/R^6 \]
**BOLTZMANN’S DISTRIBUTION**

- Probability of system being at position $x$ is
  \[ P(x) = \exp\left(-U(x)/kT\right) / Q. \]
  $U(x)$ is Potential Energy at position $x$.

- Find $Q$, the “Partition Function”, so total probability is 1.
  \[ Q = \sum \exp\left(-U(x)/kT\right) \]
Connection micro/macroscopic: intuitive view

\[ \langle O \rangle = \frac{1}{Z} \sum_i O_i e^{-\beta E_i} \]

Where \( Z = \sum_i e^{-\beta E_i} \) is the partition function

- \( E_1, P_1 \sim e^{-\beta E_1} \)
- \( E_2, P_2 \sim e^{-\beta E_2} \)
- \( E_3, P_3 \sim e^{-\beta E_3} \)
- \( E_4, P_4 \sim e^{-\beta E_4} \)
- \( E_5, P_5 \sim e^{-\beta E_5} \)
Central Role of the Partition function

The determination of the macroscopic behavior of a system from a thermodynamical point of view is tantamount to computing a quantity called the **partition function**, \( Z \), from which all the properties can be derived.

\[
Z = \sum_i e^{-\beta E_i}
\]

\[\langle O \rangle = \frac{1}{Z} \sum_i O_i e^{-\beta E_i}\]

\[\langle E \rangle = -\frac{\partial}{\partial \beta} \ln(Z) = U\]

\[p = kT \left( \frac{\partial \ln(Z)}{\partial V} \right)_{N,T}\]

\[A = -kT \ln(Z)\]

Expectation Value

Internal Energy

Pressure

Helmoltz free energy
Computation of the Partition function

The partition function is a very complex function to compute, and, in most cases, only numerical approximations are possible

\[
Z = \sum_i e^{-\beta E_i} \quad 1)
\]

Numerical approximations require:

1) the computation of the energy of the system for microstate \(i\)
   - performed using semi-empirical force fields
     CHARMM / Amber / Gromos / ...

2) a method to sample all (or a representative portion) of the microstates accessible to the system in a given macroscopic state, i.e:
   - microcanonical sampling for fixed \(N,V,E\) systems
   - canonical sampling for fixed \(N,V,T\) systems
   - isothermic-isobaric sampling for fixed \(N,P,T\) systems
   - ...

\[
2)
\]
The actual transition from State A to B is very quick (a few picoseconds).

What takes time is the waiting. The average wait before going from A to B is:

\[ \tau_{A \rightarrow B} = \frac{h}{k_b T} \exp \left[ + \frac{\Delta G}{k_b T} \right] \text{, where } \Delta G = (G_T - G_A) \]

\[ \left( \frac{h}{k_b T} \right) \sim 0.16 \text{ picoseconds at } T = 300^\circ K (27^\circ C) \]

h is Planck's constant, \( k_b \) is Boltzmann's constant
Introduction & historical note

Theoretical milestones:

Newton (1643-1727): Classical equations of motion: \( F(t) = m \, a(t) \)
Schrödinger (1887-1961): Quantum mechanical equations of motion:
\[-i\hbar \frac{\delta}{\delta t} \psi(t) = H(t) \, \psi(t)\]
Boltzmann (1844-1906): Foundations of statistical mechanics

Molecular dynamics milestones:

Metropolis (1953): First Monte Carlo (MC) simulation of a liquid (hard spheres)
Wood (1957): First MC simulation with Lennard-Jones potential
Alder (1957): First Molecular Dynamics (MD) simulation of a liquid (hard spheres)
Rahman (1964): First MD simulation with Lennard-Jones potential
Karplus (1977) & McCammon (1977)
Karplus (1983): The CHARMM general purpose FF & MD program
Kollman (1984): The AMBER general purpose FF & MD program
Car-Parrinello (1985): First full QM simulations
Kollmann (1986): First QM-MM simulations
Figure 6.7 (a) Stereoscopic view of the water molecules lying near the anionic oxygens of $g,t$ DMP after $5 \times 10^5$ steps (top); (b) same as (a) after $7.5 \times 10^5$ steps (centre); (c) same as (b) with a different viewpoint and all the water molecules included
Figure 6.4 Comparison between simulated and experimental O–O radial distribution functions of liquid water (from Lie et al. with permission [12]).
Fig. 8. Non-periodic methods for computing long-range Coulomb forces.
Treatment of the solvent contribution

1) Explicit water molecule model: TIP3P, ...

2) Implicit solvent model:

- Based on Poisson-Boltzmann Equation:
  \[ \nabla (\varepsilon(r) \nabla \phi(r)) = \rho_{\text{Macro}}(r) + \sum_i q_i n_i^0 \exp(-\beta q_i \phi(r)) \]

- or an approximation...
  - ACE potential (Schaeffer & al.)
  - SASA potential (Caflish & al.)
  - EEF1 potential (Lazaridis & al.)

For a discussion of theoretical aspects of implicit solvent models, see Roux & Simonson (*Biophys. Chem.* 1999, 78:1-20)
Fig. 4. An illustration of the Double Dynamic Programming algorithm applied to the sequence threading problem. A sequence of amino acids A–H (one letter code) is being threaded onto coordinate positions 1–9 of a structural template. Given the proposed equivalence that residue D lies on position 6, a matrix L of the scores of all other equivalences can be constructed. A best path (or alignment) is then found through this by application of the standard Dynamic Programming algorithm. The overall score for this alignment (which is an indication of how well residue D fits on position 6) is recorded in the matrix H. All potential equivalences are evaluated in this way (filling the matrix H with values) and the best consistent selection of these is found by application of the Dynamic Programming algorithm to the H matrix. This double application of the alignment algorithm at two levels gives rise to the name.
Rosetta Uses a Fragment Library + Monte Carlo Search

Examples of the best-center cluster found by Rosetta for some test proteins. In many cases the overall fold is predicted well enough to be recognizable. However, relative positions of the secondary structure elements are almost always shifted somewhat from their correct values.
AlphaFold2 Structure Predictions from CASP14